Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2013

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 13-094

MAS3CD-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	11	11	100	100	$66\frac{2}{3}$
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(7 marks)**

Students are entered into a mathematics competition each year and may be awarded a distinction, credit or participation certificate.

When the students are in Year n, the number of distinctions, credits and participation certificates is denoted d_n , c_n and p_n respectively.

Performance in successive years can be determined using a transition matrix so that

$$\begin{bmatrix} d_{n+1} \\ c_{n+1} \\ p_{n+1} \end{bmatrix} = \begin{bmatrix} 0.80 & 0.35 & 0.10 \\ 0.15 & 0.50 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} \begin{bmatrix} d_n \\ c_n \\ p_n \end{bmatrix}$$

(a) What percentage of students who obtain a credit in Year n also receive a credit in Year $(n+1)$? (1 mark)

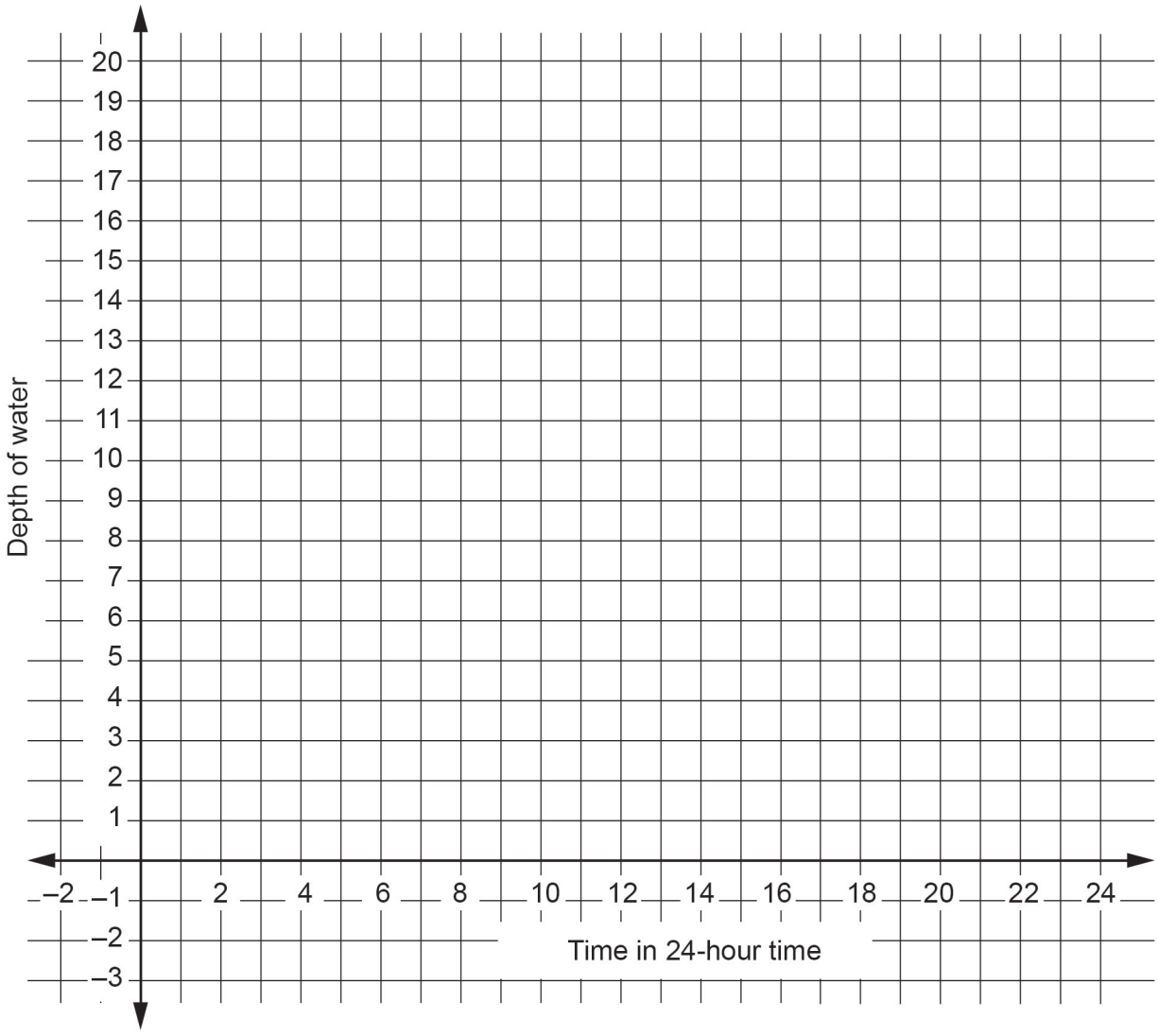
(b) In Year 8, there were 50 distinction, 41 credit and 28 participation certificates awarded. Predict the number of each type of award achieved by this group of students in Year 12. (3 marks)

(c) How many distinctions were awarded to this group in Year 7? (3 marks)

Question 10 **(12 marks)**

The depth of water in Shepherd's Harbour exhibits simple harmonic motion about a mean level. The low tide depth of 4 metres is recorded at midnight and the next high tide depth of 11 metres occurs at 6.00 am.

(a) Sketch the graph of the depth of water in Shepherd's Harbour over a complete 24 hour day (starting at midnight). (4 marks)



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(b) A container ship named *Violet*, requires a depth of 7 metres of water for safe entry into the harbour, for unloading at the dockside and for leaving again.

(i) Determine the times of the day, correct to the nearest minute, between which it is safe for *Violet* to be in the harbour. (4 marks)

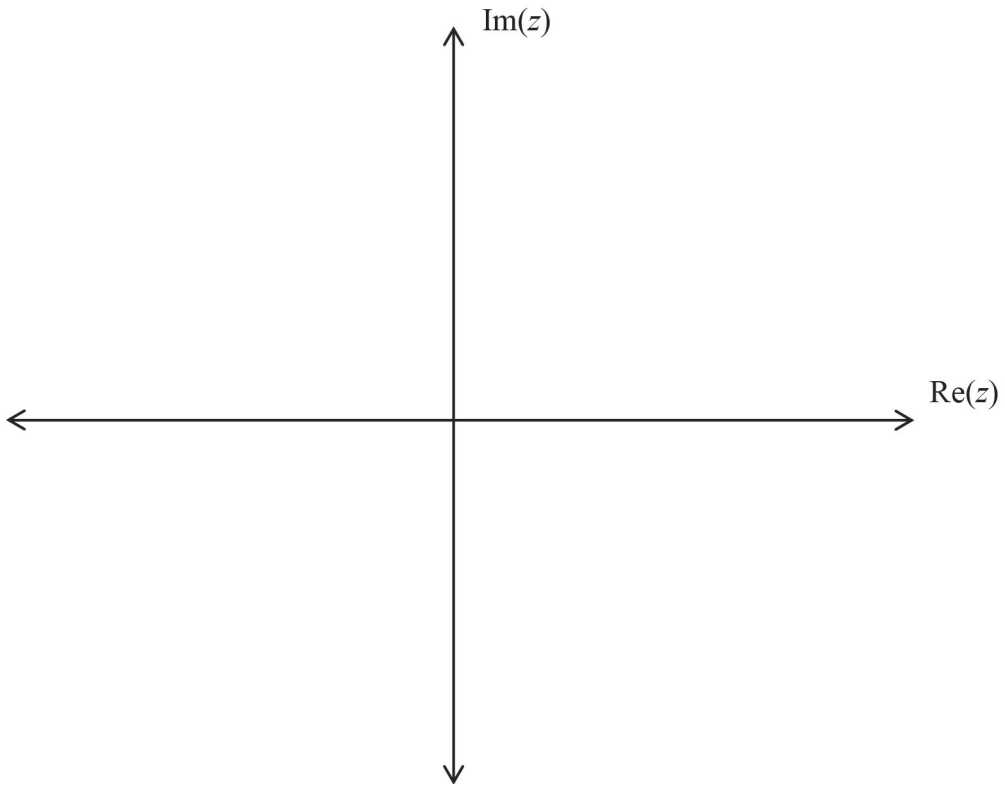
(ii) For what percentage of the day can *Violet* be safely in the harbour? (1 mark)

(c) Another ship, *Elizabeth*, requires a depth of at least d metres of water for safe operation. If *Elizabeth* can use the harbour for at least 80% of the day, determine the maximum possible value of d. (3 marks)

See next page

Question 11 **(6 marks)**

(a) Sketch in the complex plane, the region defined by $1 \leq |\, z - 8 - 6i \,| \leq 4$. (3 marks)



(b) Determine in polar form $r\text{cis}\,\theta$, $-\pi < \theta \leq \pi$, the complex number z that satisfies $|\, z - 8 - 6i \,| = 4$ and has the minimum argument. (3 marks)

See next page

Question 12 **(8 marks)**

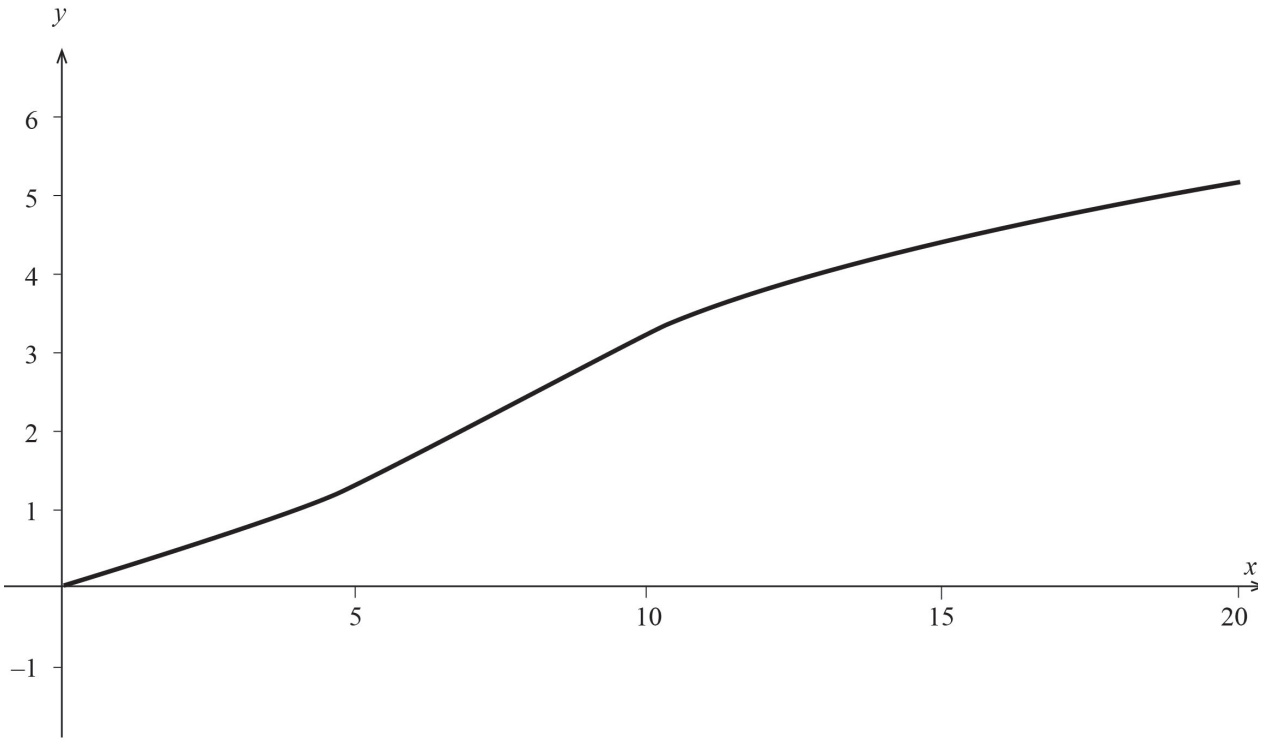
A submarine has position vector $\begin{pmatrix} -3 \\ -1 \\ -10 \end{pmatrix}$ km and needs to set a course to an underwater cave located at $\begin{pmatrix} -39 \\ -68.5 \\ -23.5 \end{pmatrix}$ km.

(a) Determine the constant velocity that the submarine should travel if it is to reach the cave after 4.5 hours. (3 marks)

(b) If there is an underwater current of $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ km/h and the submarine maintains the above velocity relative to the water, determine the closest distance between the submarine and the cave. When does this occur? (5 marks)

Question 13 **(11 marks)**

Consider the curve defined parametrically as $x = t^2$ and $y = t - \sin(t)$, $t \geq 0$.
The graph of this curve is shown below.



(a) Determine $\frac{dy}{dx}$ in terms of t and use this to find the equation of the tangent at the point on the curve where $x = 16$. Show that the equation of the tangent $y = mx + c$ has values of $m = 0.21$ and $c = 1.45$ correct to two decimal places. (5 marks)

See next page

(b) Show that $\int y\,dx = \int \left(2t^2 - 2t\sin t\right) dt$ and use this to determine the area of the region bounded by the curve, the x-axis and the tangent found in part (a). (6 marks)

See next page

Question 14 **(6 marks)**

Two of the roots of the equation $z^6 + 8i = 0$ lie in the second quadrant of the complex plane (that is where $\text{Re}(z) < 0$ and $\text{Im}(z) > 0$).

If these two roots are represented by A and B in the complex plane, determine the complex number represented by the midpoint of AB. Give your answer in Cartesian form $a + bi$ with exact expressions for a and b.

See next page

Question 15 **(8 marks)**

A plane contains the point $(5, -7, 2)$ and has a normal parallel to the vector $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$.

(a) Determine a vector equation of the plane. (2 marks)

(b) Where does the straight line $= \begin{pmatrix} -10 \\ 4 \\ -9 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -6 \end{pmatrix}$ meet the plane? (4 marks)

(c) What is the angle between the line and the plane? (2 marks)

See next page

Question 16 **(11 marks)**

The triangle OAB is defined by the points $O(0,0)$, $A(5,0)$ and $B(4,3)$.

(a) Determine the coordinates of the points O', A', B' of the triangle when it is transformed by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Describe this transformation geometrically. (3 marks)

(b) The triangle $O'A'B'$ is then transformed by a second matrix that represents a shear of factor 4 parallel to the x-axis. Write the combined effect of the two transformations as a single matrix. (3 marks)

See next page

(c) The original triangle OAB is transformed by a dilation of factor 3 parallel to the x-axis and a factor k, $k > 0$, parallel to the y-axis. If the resulting image has an area of 90 square units, determine the value of k. (3 marks)

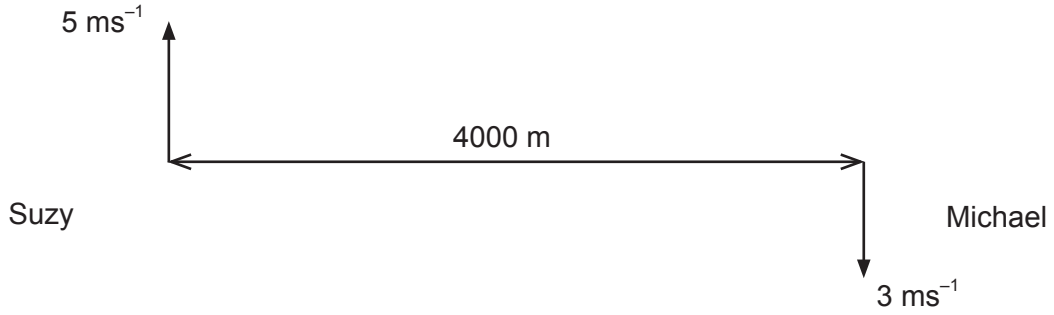
(d) Determine the matrix for the transformation that will return the image in part (c) back to the original triangle OAB. (2 marks)

See next page

Question 17 **(8 marks)**

Suzy and Michael are each riding bikes. Initially they are separated by 4000 metres and Suzy is west of Michael. Suzy starts riding north at a rate of 5 ms^{-1} and 7 minutes later Michael starts riding south at 3 ms^{-1}. At what rate is the distance between them changing 15 minutes after Suzy starts riding?

Page 15 has been left blank for you to continue your working, if necessary.



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Question 18 **(15 marks)**

The concentrations $x\,(t)$ and $y\,(t)$ of two chemicals in a reaction are known to decay according to the pair of differential equations

$$\frac{dx}{dt} + 2x - y = 0 \quad \text{and} \quad \frac{dy}{dt} - x + 2y = 0$$

subject to initial conditions $x\,(0) = 1$ and $y\,(0) = 4$.

Let $z_1 = x + y$ and $z_2 = x - y$.

(a) Show that $\frac{dz_1}{dt} = -z_1$ and $\frac{dz_2}{dt} = -3z_2$. (4 marks)

(b) By solving these differential equations for z_1 and z_2 deduce the expressions for $x\,(t)$ and $y\,(t)$. (5 marks)

(c) Of importance in the chemical process is the ratio of the two concentrations, $r(t) = \dfrac{y(t)}{x(t)}$.

(i) Show that $r(t) = \dfrac{5 + 3e^{-2t}}{5 - 3e^{-2t}}$. (2 marks)

(ii) Prove that $r(t)$ decreases for all values of t. (2 marks)

(iii) What happens to $r(t)$ as $t \to \infty$? (2 marks)

See next page

Question 19 **(8 marks)**

The function $f(n)$ is defined for integer values n as follows:

$$f(n) = 1$$
$$f(2n) = f(n)$$
$$f(2n + 1) = (f(n))^2 - 2.$$

Determine $f(1) + f(2) + f(3) + \ldots + f(2013)$.

End of questions

Additional working space

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